Unique expectations and pseudo-expectations for abelian C^*-inclusions

Abstract: By the Gelfand-Naimark Theorem, every unital abelian C^*-algebra $\mathcal{A}$ is $*$-isomorphic to $C(X)$, the continuous complex-valued functions on a compact Hausdorff space X. In principle, any property of $\mathcal{A}$ corresponds to a property of X. For example:

- $\mathcal{A}$ is simple $\iff$ X is a singleton;
- $\mathcal{A}$ is projectionless $\iff$ X is connected;
- $\mathcal{A}$ is separable $\iff$ X is metrizable;
- $\mathcal{A}$ is injective $\iff$ X is extremally disconnected (Stonean).

Now let $\mathcal{D} \subseteq \mathcal{A}$ be an inclusion of unital abelian C^*-algebras. If $\mathcal{D} \cong C(X)$ and $\mathcal{A} \cong C(Y)$, then the corresponding inclusion $\iota : C(X) \to C(Y)$ is given by $f \mapsto f \circ j$, where $j : Y \to X$ is a continuous surjection. In this talk, we characterize when $\mathcal{D} \subseteq \mathcal{A}$ admits a unique conditional expectation (resp. a unique pseudo-expectation) in terms of properties of j. Namely:

- $\mathcal{D} \subseteq \mathcal{A}$ admits a unique conditional expectation $\iff$ there exists a unique G_δ set $A \subseteq Y$ such that $j|_A : A \to X$ is an open surjection.
- $\mathcal{D} \subseteq \mathcal{A}$ admits a unique pseudo-expectation $\iff$ there exists a unique minimal closed set $K \subseteq Y$ such that $j|_K : K \to X$ is a surjection.

The first result is only valid if $\mathcal{A}$ is separable, while the second result is true in general. The second result is joint work with David Pitts.